

# PADLOCK PROBLEM

If we have a standard tumbler style padlock that when locked never shows any numbers correctly how many times would we need to view the padlock and note down the numbers that it isn't until we managed to determine what the code is.

To do this we need to look at the cumulative distribution of a discrete process which is;

$$P(N \leq n) = 1 - \frac{1}{m^n} \sum_{k=1}^m (-1)^{k+1} \binom{m}{k} (m-k)^n$$

See <https://math.stackexchange.com/questions/379525/probability-distribution-in-the-coupon-collectors-problem> (This is a similar class to the coupon collectors problem)

This gives us the distribution of how many times we need to look to get one tumbler right however we have four in our case but for  $n_{\text{tumb}}$  tumblers each with  $m$  numbers on their dials we have the waiting time for the complete combination to be revealed as;

$$P(N \leq n) = \left( 1 - \frac{1}{m^n} \sum_{k=1}^m (-1)^{k+1} \binom{m}{k} (m-k)^n \right)^{n_{\text{tumb}}}$$

Where  $m$  is number of digits -1 and  $n$  is the number of times looked.

So for our padlock,  $m=9$ ,  $n_{\text{tumb}}=4$ .

We get that the median is 34 (this is the first number where the CDF > 0.5) the waiting time to get it 99% of the time is 70 and you would get the code in 20 tries 1.7% of the time.

The mean is calculated by summing the fractions of times it isn't solved by a particular number of views;

$$\mu = \sum_{i=0}^{\infty} (1 - P(N \leq i)) = 36.276$$

We only have to sum to 100 because  $P(N \leq n)$  for  $n > 100$  is practically 0. This nicely matches the numerical data put together by Joel Uckleman . Many thanks to Mark Manzocchi of SNC Lavalin as he did most of the heavy lifting on this

